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TERM END EXAMINATIONS (TEE) – August 2024

Programme	: B.Tech.	Semester	: Fall Semester 2024-25
Course Title	: Applied Linear Algebra	Course Code	: MAT3002
Date/Session	: 28 Aug 2024/Session-I	Slot	: B22+B23+E21+E22
Time	: 3 Hrs.	Max. Marks	: 100

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	Find the all values of k for which the system has (a) no solution, (b) infinitely many solutions $$x+z=k^2$$ $$2x+y+3z=-3k$$ $$3x+y+4z=-2$$	8
	(b)	Find the row Echelon form of $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 2 & 2 \\ 6 & -3 & 1 & 4 \end{bmatrix}$	4
		OR	12
	(c)	Determine an LDU decomposition of the matrix $$A=\begin{bmatrix} 1 & 3 & -1 \\ 2 & 8 & 4 \\ -1 & 3 & 4 \end{bmatrix}$$	
2	(a)	Let $W_1=\{(x,y,z,w): 3x+y-7w=0\}$ and $W_2=\{(x,y,z,w): -2y+5z-w=0\}$. Find the dimension of W_1 and W_2, then find the basis for W_1+W_2 and $W_1\cap W_2$.	12
		OR	
	(b)	Show that the vectors $\alpha_1=(1,1,0,0), \alpha_2=(0,0,1,1), \alpha_3=(1,0,0,4), \alpha_4=(0,0,0,2)$ form a basis for R^4. Find the coordinates of each of the standard basis vectors in the ordered basis $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$.	12
3	(a)	Determine whether the given transformation $T: R^3 \to R^3$ is invertible or not, where $T(x,y,z)=(x+3y-2z, 2x+3y, y-z)$. If it is invertible then find T^{-1}.	12
		OR	
	(b)	Consider the following ordered bases of R^3, B_1 be the standard basis and $B_2=\{(1,1,1),(1,1,0),(1,0,0)\}$. Let $T(x,y,z)=(2y+x, x-4y, 3x)$ be the linear transformation and Q be the transition matrix from B_2 to B_1. Show that $[T]_{B_2}=Q^{-1}[T]_{B_1}Q$	12
4	(a)	Find out that $(R^2,\langle .,.\rangle)$, where $\langle x,y\rangle = x_1y_1-x_1y_2-x_2y_1+3x_2y_2$ is an inner product space or not.	6

(b) Consider the space $C[0,1]$ with the inner product defined by $\langle f,g\rangle = \int_0^1 f(x)g(x)dx$. Find the angle between $f(x) = x^2$ and $g(x) = x^4$. 6

OR

(c) Let U be the subspace of $V = R^3$ spanned by $\{(2,4,4),(-2,0,4)\}$. Find the orthonormal basis for U and extend this basis to obtain the orthonormal basis for V. 12

5 (a) Find the singular value decomposition of $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. 12

OR

(b) Find the QR factorization of the following matrix: 12

$$A = \begin{bmatrix} 1 & 3 \\ -2 & -1 \\ 3 & 3 \end{bmatrix}$$

PART B – (40 Marks)

6 Find the inverse of A by Gauss-Jordan method, where 8

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 3 & 4 & 0 \end{bmatrix}$$

7 Let the vector space R^4 be equipped with the Euclidean inner product. For which value(s) of k are $\boldsymbol{u}$ and $\boldsymbol{v}$ orthogonal? 8

a). $\boldsymbol{u} = (2,1,3), \boldsymbol{v} = (1,7,k)$

b). $\boldsymbol{u} = (k,k,1), \boldsymbol{v} = (k,5,6)$

8 Find the linear transformation T which maps $(1,0,1)$ to $(1,2,1)$, $(2,1,-1)$ to $(1,1,1)$ and $(1,3,0)$ to $(1,0,1)$. 8

9 Find the orthogonal complement of the subspaces S of R^4 spanned by the two column vectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$. 8

10 Find the least square solution x in R^2 of $Ax = b$, where 8

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ -1 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$