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MID TERM EXAMINATIONS – October-November 2023

Programme	:	**B.Tech.**	Semester	:	**Fall Semester 2023-24**
Course Title/ Course Code	:	**Calculus and Laplace Transform / MAT1001**	Slot	:	**C11+C12+C13**
Time	:	**1 ½ hours**	Max. Marks	:	**50**

Answer all the Questions

Q.No.	Sub. Sec.	Question Description	Marks
1	(a)	Let $f(x,y) = \dfrac{x^2y^2}{x^2y^2+(x^2-y^2)^2}$ for $(x,y) \neq (0,0)$. Show that the iterated limits $lim_{y\to 0}(lim_{x\to 0} f(x,y))$ and $lim_{x\to 0}(lim_{y\to 0} f(x,y))$ exist, but $lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.	5
	(b)	Expand e^{xy} in powers of $(x+1)$ and $(y+1)$ up to second degree terms using Taylor's series.	5
2		A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point *(x, y, z)* on the probe's surface is $T(x,y,z) = 8x^2 + 4yz - 16z + 600.$ Find the hottest point on the probe's surface.	10
3		Calculate the volume of the solid region D bounded above by the plane $z = \sqrt{3}$ and below by the sphere $x^2 + y^2 + z^2 = 4$ by using the concept of integrals.	10
4		Draw the region of integration and evaluate $\int_{-1}^{0}\int_{-x}^{1+\sqrt{1-x^2}} \frac{1}{(x^2+y^2)^2}\, dy\, dx$.	10
5		Is $\vec{F} = (y^2 \cos x + z^3)\hat{\imath} + (2y \sin x - 4)\hat{\jmath} + (3xz^2 + 2)\hat{k}$ is conservative field ? If so, find scalar potential of $\vec{F}$.	10

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