TERM END EXAMINATIONS (TEE) – December 2023- January 2024

Programme	:	**B.Tech.**	**Semester**	:	**Fall 2023-24**
Course Title/ Course Code	:	**Calculus and Laplace Transform/ MAT1001**	**Slot**	:	**F11+F12+F14**
Time	:	**3 Hrs.**	**Max. Marks**	:	**100**

(Answer all the Questions)

Q.No		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	Prove $u = x^3 + y^3 - 63(x + y) + 12xy$ *has maximum at* $(-7,-7)$ and minimum at (3,3).	6
	(b)	If $U = e^{xyz}$, find the value of $\frac{\partial^3 U}{\partial x \partial y \partial z}$.	6
		OR	
	(c)	A scope probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth atmosphere and its surface begins to heat. After one hour the temperature at the point (x, y, z) on the surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe surface.	12
2	(a)	Changing the order of integration of $\int_0^\infty \int_0^\infty e^{-xy} sinnx dxdy$ show that $\int_0^\infty \frac{sinnx}{x} dx = \frac{\pi}{2}$.	6
	(b)	Evaluate $\int_0^1 \int_0^x (x^2 + y^2) dA$ where dA indicated small area in xy plane.	6
		OR	
	(c)	Find the volume of the cylindrical column standing on the area common to the parabolas $y^2 = x, x^2 = y$ and cut off by the surface $z = 12 + y - x^2$.	12
3	(a)	Evaluate $\iint_S (yzi + zxj + xyk).\overrightarrow{dS}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.	12

OR

(b) Verify Green's theorem in the plane $\int_C (3x^2 - y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by $x \geqslant 0$; $y \geqslant 0$ and $2x - 3y = 6$. 12

4 (a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2$. 6

(b) Solve: $\frac{dy}{dx} + y cot x = 4x cosec x$, given that $y(\frac{\pi}{2})=0$. 6

OR

(c) An inductor of $L = 2\,heneries$, resistor $R = 16\,ohms$ and capacitor $C = 0.02\,farads$ are connected in series with a battery of e.m.f. $E = 100 sin\,3t$. At $t = 0$, the charge on the capacitor and current in the circuit are zero. Find the charge and current at $t > 0$. Where the model of the differential equation is 12

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E.$$

5 (a) Solve using Laplace transform 12

$$2\frac{d^2y}{d^2x} + 3\frac{dy}{dx} - 2y = te^{-2t}, \qquad given\ y(0) = 0;\ y'(0) = -2.$$

OR

(b) Find the inverse Laplace transform of $f(s) = log(\frac{s+a}{s+b})$. 6

(c) Find the Laplace transform of $\frac{1-cos2t}{t}$. 6

PART B – (40 Marks)

6 Find the tangent plane to the surface $z = x\,cosy - ye^x$ at (0,0,0). 8

7 Evaluate the following integral by changing the order of the integration 8

$$\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}} dydx.$$

8 Find the directional derivative of $U = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $(2, -1, -2)$. 8

9 Solve: $(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 2sin(2ln(x+1))$, $x > -1$. 8

10 Using convolution theorem find $L^{-1}[\frac{1}{(s^2+1)(s+1)}]$. 8

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