TERM END EXAMINATIONS (TEE) – December 2023- January 2024

Programme	: B.Tech.	Semester	: Fall 2023-24
Course Title/ Course Code	: Calculus and Laplace Transform/ MAT1001	Slot	: C11+C12+C13
Time	: 3 Hrs.	Max. Marks	: 100

Answer ALL the Questions

Q. No.	Question Description	Marks
	PART A – (60 Marks)	
1	(a) If $z = f(x,y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, show that $$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2y}\frac{\partial z}{\partial y}$$	6
	(b) Expand $\sin xy$ in powers of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$ as far as terms of second degree.	6
	OR	
	(c) Determine the critical points and locate any relative minima, maxima and saddle points of function f defined by $$f(x,y) = 2x^2 - 4xy + y^4 + 2$$	12
2	(a) Evaluate $\iint_D (y^2 + 3x)dA$ where D is the region between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.	12
	OR	
	(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.	12
3	(a) Using Gauss divergence theorem evaluate $\iint_S \vec{F}.\hat{n}\, dS$ where $\vec{F} = (x+z)\hat{\imath} + (y+z)\hat{\jmath} + (x+y)\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane bounded by xy-plane.	6
	(b) Use Green's Theorem to evaluate $\oint xy\, dx + x^2y^3\, dy$, where C is the triangle with vertices (0, 0), (1, 0), and (1, 2) with positive orientation.	6
	OR	
	(c) Evaluate $\iint_S \vec{F}.\hat{n}\, dS$ where $\vec{F} = -x\hat{\imath} + 2y\hat{\jmath} - z\,\hat{k}$ and S is the portion of $y = x^2 + z^2$ that lies behind $y = 1$ oriented in the positive y –axis.	12
4	(a) At $t = 0$ a current of 2 amperes flows in an LCR circuit with resistance R = 3 ohms, inductance L= 0.1 henrys, and capacitance C = .01 farads. Find the current flowing in the circuit at t > 0 if the initial charge on the capacitor is 0 coulomb. Assume that E(t) = 0 for t > 0. The model of the differential equation is $$Lq'' + Rq' + \frac{1}{C}q = E(t) \qquad q(0) = q_0, \qquad q'(0) = I_0$$	6

(b) Find the general solution of the following differential equation using method of undetermined coefficients 6

$$\frac{d^2y}{dx^2} - 4y = 2e^{2x}.$$

OR

(c) Find the general solution of the following differential equation using method of variation parameters 12

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x^2+1}$$

5 (a) Solve using Laplace transform 12

$$y'' + 2y' = t\,e^{-t}, \quad y(0) = 6, y'(0) = -1$$

OR

(b) Find the Laplace transform of 6

$$f(t) = \frac{\cos 4t - \cos 5t}{t} + te^{2t}\sin 3t$$

(c) Find the inverse Laplace transform of 6

$$H(s) = \frac{se^{-s}}{(3s+2)(s-2)}$$

PART B – (40 Marks)

6 Show that the equation of the tangent plane to the ellipsoid 8

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at the point (x_0, y_0, z_0) can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1$$

7 Evaluate the following integral by changing the order of integration 8

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{2x}{\sqrt{x^2+y^2}}\,dydx$$

8 Find the directional derivative of the function $f(x, y, z) = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q has coordinates $(5, 0, 4)$. In what direction, it will be maximum and what is its value? 8

9 Find the general solution of the following Cauchy-Euler equation 8

$$x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 16y = x.$$

10 Find the Laplace transform of the following periodic function 8

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 2-t & 1 \le t < 2 \end{cases} \qquad f(t+2) = f(t).$$

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