TERM END EXAMINATIONS (TEE) – July 2024

Programme	:	B.Tech.	Semester	:	Winter Semester 2023-24
Course Name/ Course Code	:	Differential and Difference Equations/ MAT2001	Slot	:	C12+C13
Time	:	3 Hrs.	Max. Marks	:	100

Answer ALL the Questions

Q. No.		Question Description	Marks

PART A – (60 Marks)

1 (a) Show that matrix A is similar to matrix B, **12**

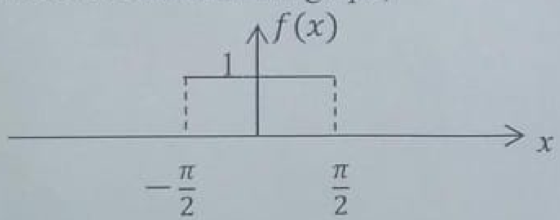
Where, $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

OR

(b) Solve the initial-value problem **12**

$$\frac{dX}{dt} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} X,\ X(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \text{ where, } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

2 (a) Showing the details of your work, find the Fourier coefficients of the given function $f(x)$, which is shown in the graph, **12**



Also prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} +, \dots, = \frac{\pi^2}{8}$ by using Parseval's identity.

OR

(b) Find the Fourier series of the given function **12**

$f(x) = 3x + 2\pi \quad$ if $\ -\pi < x < \pi$

and $f(x + 2\pi) = f(x)$.

Also find the amplitude and phase of the above problem.

3 (a) Find the temperature distribution in semi-infinite bar with its end point and lateral surface insulated and with initial temperature distribution in the bar is prescribed by $f(x)$. Deduce the solution when $f(x) = e^{-ax}$. **12**

OR

(b) Represent $f(x)$ as an exponential Fourier transform when 12

$$f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & otherwise \end{cases}$$

Show that the result can be written as

$$f(x) = \frac{1}{\pi}\int_0^\infty \frac{\cos\alpha x + \cos\alpha(x-\pi)}{1-\alpha^2}\, d\alpha.$$

4 (a) Find the inverse Z-transform of $\frac{Z}{Z^3-Z^2+Z-1}$. 12

OR

(b) Using the convolution theorem, find the inverse Z-transform of $\left(\frac{Z}{Z-a}\right)^3$. Also deduce for $\left(\frac{Z}{Z-1}\right)^3$. 12

5 (a) Solve the difference equation using undetermined coefficients method 12

$$u_{n+3} - 12u_{n+2} + 48u_{n+1} - 64u_n = 5.4^n$$

OR

(b) Use Z-tramsform, solve the difference equation 12

$u_{n+2} - 2u_{n+1} + u_n = 3n + 5.$

PART B – (40 Marks)

6 Prove that the give matrix is diagonalize 8

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

7 Find the Fourier cosine series and Fourier sine series. Sketch $f(x)$ and its two periodic extensions. (Show the details of your work.) 8

$f(x) = \pi - x, \quad 0 < x < \pi$.

8 Find the inverse Fourier sine transform of $\frac{1}{s}e^{-as}$. 8

9 Prove that $Z(a^n \sin n\theta) = \frac{aZ\sin n\theta}{Z^2-2aZ\cos\theta+a^2}$, by using Z-transform. 8

10 Solve the difference equation by using method of undetermined coefficients 8

$y_{n+2} - 5y_{n+1} + 6y_n = 2\,n^2 - 6n - 1.$

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