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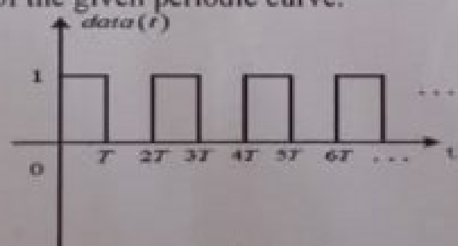
TERM END EXAMINATIONS (TEE) – May 2024

Programme	:	B.Tech.	Semester	:	Winter Semester 2023-24
Course Name/ Course Code	:	Differential and Difference Equations/ MAT2001	Slot	:	D23+D24
Time	:	3 Hrs.	Max. Marks	:	100

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	Diagonalize the matrix $A = \begin{bmatrix} 6 & -3 & 0 & 9 \\ 0 & 4 & 1 & -5 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.	12
		OR	
	(b)	Solve the given system of first order ordinary differential equations $\frac{dx}{dt} = -3x - 2y + 2z, \frac{dy}{dt} = 2x + y - 2z$ and $\frac{dz}{dt} = -2x - 2y + z$ by using eigen value method.	12
2	(a)	Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier expression of $f(x)$ and also deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$	12
		OR	
	(b)	Find the Fourier series of the given periodic curve. 	12

Q. No.		Question Description	Marks
3	(a)	Find the Fourier transform of $f(x) = \begin{cases} 1, & \lvert x \rvert < 1 \\ 0, & \lvert x \rvert > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$.	6
	(b)	Find Fourier cosine transform of e^{-x^2}.	6
		OR	
	(c)	Using Parseval's identities prove that $\int_0^\infty \frac{t^2}{(t^2+1)^2}\, dt = \frac{\pi}{4}$.	6
	(d)	Use convolution theorem to find the inverse Fourier transform of $\frac{1}{(1+s^2)^2}$, given that $\frac{2}{1+s^2}$ is the Fourier transform of $e^{-\lvert x \rvert}$.	6

4 (a) Find the Z-transform of $3n^2 - \frac{\sin n\frac{\pi}{2}}{4} + 5$. 6

(b) Determine f_0, f_1, f_2 when $Z\{f(n)\} = F(z) = \frac{(z-1)^2(z+2)}{(z+3)(z+5)^2}$ 6

OR

(c) Find Z-transforms of $n^2 e^{n\theta} u(t)$, where $u(t)$ is an unit step function. 6

(d) Show that $Z\left(\frac{1}{n!}\right) = e^{1/z}$ and also evaluate $Z\left(\frac{1}{(n+1)!}\right)$ and $Z\left(\frac{1}{(n+2)!}\right)$. 6

5 (a) Solve the following difference equation 12
$9y(n+2) - 6y(n+1) + y(n) = n$ with $y(0) = 1$ and $y(1) = 1$.

OR

(b) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 3^n$ with $y_0 = y_1 = 0$ using Z-transforms. 12

PART B – (40 Marks)

6 If eigen values of $4 X 4$ order matrix A are $1, 1, 0, 0$ then find eigen valus of 8

i) A^2,
ii) $A - 2I$,
iii) Determinant of A,
iv) Adjoint of A,
v) A^{-1},
vi) 3A,
vii) Trace of A and
viii) Determinant of $A^2 + 3A + 7$.

7 Find Fourier sine series of x over the interval $(0, \pi)$. 8

8 Find Fourier sine transform of $\frac{e^{-ax}}{x}$. 8

9 Find the inverse Z-transform of $\frac{3z}{(z+2)(z-4)}$ using convolution theorem. 8

0 Solve the difference equation $6y_{n+2} - y_{n+1} - y_n = 0$ with $y_0 = y_1 = 1$ using Z-transforms. 8