BHOPAL

www.vitbhopal.ac.in

TERM END EXAMINATIONS (TEE) – May 2024

Programme	: **B.Tech.**	Semester	: Winter Semester 2023-24
Course Title/ Course Code	: **Differential And Difference Equations/ MAT2001**	Slot	: A21+A22
Time	: **3 Hrs.**	Max. Marks	: 100

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	Use matrix method to find the solution of the system of differential equations: $$\begin{aligned}\frac{dx}{dt}&=-5x+y+6e^{2t}\\ \frac{dy}{dt}&=4x-2y-e^{2t},\end{aligned} \quad \text{with } x(0)=1,\ y(0)=2.$$	12
		OR	
	(b)	Check whether the matrix $M=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable. If so, find an invertible matrix P such that $P^{-1}MP$ is diagonal.	12
2	(a)	Find the Fourier series expansion for the function $$f(x)=\begin{cases} 0, & -\pi \le x < 0 \\ \frac{1}{4}\pi x, & 0 < x < \pi. \end{cases}$$ Deduce that: $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots$.	12
		OR	
	(b)	Find the half range Fourier Sine and Cosine series of the function: $$f(x)=\begin{cases} \left(\frac{1}{4}-x\right), & 0 < x < \frac{1}{2} \\ \left(x-\frac{3}{4}\right), & \frac{1}{2} < x < 1. \end{cases}$$	12

3 (a) Compute the Fourier cosine integral of the function $f(x)=\begin{cases}|\sin x|, & |x|\le \pi\\ 0, & |x|\ge \pi.\end{cases}$ 12

Also deduce that: $\int_0^\infty \left(\frac{\cos\lambda\pi+1}{1-\lambda^2}\right)\cos\left(\frac{\lambda\pi}{2}\right)d\lambda=\frac{\pi}{2}.$

OR

(b) (i) Obtain the Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|>1.\end{cases}$ 8

(ii) Find the convolution of $f(x)=\sin x,\ g(x)=e^{-m|x|},\ m>0$. 4

4 (a) (i) Find the Z-transform of the sequence $f(k)=\begin{cases}7^k, & k<0\\ \frac{1}{8^k}, & k=0,2,4...\\ \frac{1}{11^k}, & k=1,3,5,...\end{cases}$ 6

(ii) Use convolution theorem to find the inverse Z-transform of the function 6

$$F(z)=\frac{8z^2}{(2z-1)(4z+1)}.$$

OR

(b) (i) Find the Z-transform of the sequence 6

$f(k)=\beta\left(-\frac{1}{\alpha}\right)^k u(k)-\alpha(\beta)^k u(-k-1),\ \alpha,\beta>0.$

(ii) Find inverse Z-transform of $F(z)=\frac{2z^2-5z}{(z-2)(z-3)},\ |z|>3.$ 6

5 (a) Solve the following difference equation using Z-transform: 12

$y_{n+2}-3y_{n+1}+2y_n=5^n u(n),\ y_0=1,\ y_1=2.$

OR

(b) Solve the following difference equation: 12

$y_{n+2}-2y_{n+1}+4y_n=2^n\left[\cos\left(\frac{n\pi}{3}\right)+\sqrt{3}\sin\left(\frac{n\pi}{3}\right)\right].$

PART B – (40 Marks)

6 Find the eigenvalues and eigenvectors of the matrix $M=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & 1\\ 1 & -1 & 2\end{bmatrix}.$ 8

7 Use Parseval's identity for the function $f(x)=x, \quad -\pi \leq x \leq \pi$, deduce that: 8

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\dots$$

8 Find the function $f(x)$, if its Fourier Cosine transform is given by 8

$$F_c(s)=\begin{cases}\frac{1}{\sqrt{2\pi}}\left(\alpha-\frac{s}{2}\right), & s<2\alpha \\ 0, & s\geq 2\alpha.\end{cases}$$

9\. If $F(z)=\frac{2z^2+5z+14}{(z-1)^4}$, then using initial value theorem of Z transform to find the values of $f(0), f(1), f(2)$ and $f(3)$. 8

10 Solve the following difference equation: 8

$$y_{n+2}-7y_{n+1}+10y_n=3e^{2n}-3^n, \quad y_0=1, y_1=3.$$

⇔⇔⇔