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TERM END EXAMINATIONS - MAY 2024

Programme	B.Tech	Semester	Winter Sem 2023-2024
Course Name/ Course Code	Differential and Difference Equation / MAT2001	Slot	A22+A23
Time	3 Hrs.	Max. Marks	100

Answer ALL the Questions

Q.No.	Sub. Sec.	Question Description	Marks
		PART-A(60 Marks)	
1.	a)	Find Eigen Values and Eigen Vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.	12
		OR	
	b)	Find the general solution of the system of differential equations $\frac{dx}{dt} = x + y,\ \frac{dy}{dt} = 4x + y.$	12
2.	a)	Draw the graph the following function and find its corresponding Fourier Series $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$	12
		OR	
	b)	Find Fourier Series of the function f(x) = x in $[-\pi, \pi]$, then apply Parseval's Identity to evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2}$.	12
3.	a)	The temperature u(x,t) at any point of an infinite bar satisfies the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 y}{\partial x^2}$, $0<x<\infty$, $t>0$ the initial temperature along the length of the bar is given by $u(x,0) = \begin{cases} 1, & \lvert x \rvert < 1 \\ 0, & \lvert x \rvert > 1 \end{cases}$ Determine the expression of u(x, t).	12
		OR	
	b)	Find Fourier Transform of $f(x) = \begin{cases} 1 - x^2, & \lvert x \rvert \le 1 \\ 0, & \lvert x \rvert > 1 \end{cases}$ and hence evaluate (a) $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$ (b) $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} dx$.	12
4	a)	Use convolution theorem to find $Z^{-1}\left[\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}\right]$.	12
		OR	
	b)	Evaluate $Z\left[\frac{1}{n!}\right]$.	6

1-n
$\left(\frac{1}{2}\right)^{2n}$ + 2
1 - 2n
-2
ICI
6 + 8

c) Evaluate $Z[(n+1)^2]$. 6

5 a) Solve $y_{n+2} + 10y_{n+1} + 25y_n = 1 + n^2$ by using method of undetermined coefficients. 12

OR

b) Solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n, y_0 = 0, y_1 = 1$ by using Z transforms. 12

PART-B(40 Marks)

6 If λ is an Eigen value of a non-singular matrix A then prove that $\frac{|A|}{\lambda}$ is an Eigen value of A^{-1}. 8

7 a) Write the Dirichlet conditions for f(x) in Fourier series 4

b) For a given $f(x) = x \sin x, 0 \leq x \leq 2\pi$, find its a_0 using Fourier series. 4

8 Apply Fourier integral to prove that $\int_0^\infty \frac{\cos\lambda x}{1+\lambda^2} dx = \frac{\pi}{2} e^{-x} (x \geq 0)$. 8

9 Find relation between Z transforms and Laplace transforms. 8

10 Solve Fibonacci relation $a_{n+2} = a_{n+1} + a_n$ with initial condition $a_0 = 0$ and $a_1 = 1$. 8