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TERM END EXAMINATIONS (TEE) –August 2024

Programme	: B.Tech.	Semester	:	Fall Semester 2024-2025
Course Title	: Differential And Difference Equations	Course Code	:	MAT2001
Date/Session	: 27 Aug 2024/Session-I	Slot	:	B11+B12+B13+E12
Time	: 3 Hrs.	Max. Marks	:	100

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	Find the general solution to the system of differential equations by using the matrix method; $y_1' = -4y_1 - 3y_2 - y_3$; $y_2' = 2y_1 + 3y_2 + 2y_3$; $y_3' = 4y_1 + 2y_2 + 3y_3$	12
		OR	
	(b)	A square matrix 'A' is defined by $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the modal matrix P such that $P^{-1}AP$ is diagonal matrix D of A.	12
2	(a)	Obtain the Fourier cosine series expansion of the periodic function defined by $f(t) = \sin\left(\frac{\pi t}{l}\right),\ 0 < t < l$	12
		OR	
	(b)	Find the Fourier half-range cosine series of the function $f(x) = \begin{cases} 2t & , 0<t<1 \\ 2(2-t), & 1<t<2 \end{cases}$.	12
3	(a)	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } \lvert x\rvert \le 1 \\ 0 & \text{if } \lvert x\rvert > 1 \end{cases}$ and use it to evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx$	12
		OR	
	(b)	If Fourier cosine transform is given by $F_c(s) = \frac{1}{2}\tan^{-1}\left(\frac{2}{s^2}\right)$, by implementing the concept of inverse transform find $f(x)$	12
4	(a)	Obtain the inverse Z-transform of $\frac{1}{(z-3)(z-2)}$, (i) $\lvert z\rvert < 2$, (ii) $2 < \lvert z\rvert < 3$, (iii) $\lvert z\rvert > 3$	12
		OR	

(b) Find $Z^{-1}\left(\frac{z^2}{(z-1)(z-2)}\right)$ using convolution theorem. 6

(c) Find the z transform of the function $f(k)=\cos ak, k \geq 0$. 6

4(a) Use Z-transform to solve the difference equation. $y_{n+2}-2y_{n+1}+y_n=3n+5$, 12
$y(0)=y(1)=0$

OR

5 (b) A seed of a particular plant produces 8-fold when one year old and produces 18-fold when two or more years old. If a_n denotes the number of seeds produced at the end of the n^{th} year, express a_n in terms of n. 6

(c) If Raju invests Rs 1000 at 6% interest compounded quarterly, how many months must he wait for his money to double (Note that Raju cannot withdraw the money before the quarter is up). How many months it trebles. 6

PART B – (40 Marks)

6 The matrix $A=\begin{bmatrix} a & h \\ h & b \end{bmatrix}$ is transformed to the diagonal form $D=T^{-1}AT$, where $T=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. Find the value of θ which gives this diagonal transformation 8

7 Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier series of $f(x)$ as given in the following tables. 8

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

8 Using Fourier integral representation, show that

$$\int_0^\infty \frac{\cos\omega x+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases} 0, & \text{if } x<0 \\ \frac{\pi}{2}, & \text{if } x=0 \\ \pi e^{-x}, & \text{if } x>0 \end{cases}$$ 8

9 Find $Z(u_{n+2})$ if $Z(u_n)=\frac{z}{z-1}+\frac{z}{z^2+1}$. Also find u_0 and u_1. 8

10 Apply the residue calculus and solve the difference equation $y_{k+2}+4y_{k+1}+3y_k=3^k$; 8
$y_0=0$ and $y_1=1$ by Z-transform.

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