TERM END EXAMINATIONS (TEE) – August 2024

Programme	:	B.Tech.	Semester	:	Fall Semester 2024-2025
Course Name	:	Differential and Difference Equations	Course Code	:	MAT2001
Date/Session	:	21 Aug 2024/Session I	Slot	:	A14+B14+E14+F14
Time	:	3 Hrs.	Max. Marks	:	100

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	The matrix A is defined as $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ Find the Eigen values and Eigen vectors of $A^2 + 5A - 2I$.	12
		OR	
	(b)	Find the matrix P such that such that $P^{-1}AP$ is a diagonal matrix, where $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	12
2	(a)	If $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 \le x \le 2\pi$ Show that $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{cosnx}{n^2}$	12
		OR	
	(b)	Find the Fourier series for the function $f(x) = \begin{cases} -1 & for\ -\pi < x < \pi/2 \\ 0 & for\ -\pi/2 < x < \pi/2 \\ 1 & for\ \pi/2 < x < \pi \end{cases}$	12
3	(a)	Find the Fourier cosine and sine transform of $f(x) = e^{-ax}$ for $x \ge 0$ and $a > 0$and hence evaluate $\int_0^{\infty} \frac{cosmx}{m^2+a^2} dm$ and $\int_0^{\infty} \frac{msinmx}{m^2+a^2} dm$	12
		OR	
	(b)	Find the Fourier transform of $f(x) = \begin{cases} 1, for\ \lvert x \rvert < a \\ 0, for\ \lvert x \rvert > a \end{cases}$ Hence evaluate $\int_0^{\infty} \frac{sinax}{x} dx$	12

4 (a) Find the Z –transform of

(i) $e^{-in\theta}$ and deduce that $Z(\cos n\theta) = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$ and $Z(\sin n\theta) = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$ 12

(ii) $3n + 6\sin\frac{n\pi}{4} - 7a^n$

OR

(b) Find the inverse Z- transform of 12

$$\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$$

5 (a) Solve the recurrence relation 12

$$a_r - 5a_{r-1} + 6a_{r-2} = 1 \text{ ,with initial condition } a_0 = 0, a_1 = 1$$

OR

(b) Find the general solution of $(E^2 + 4)y_n = \cos n\pi$ 12

PART B – (40 Marks)

6 Apply Eigen values and Eigen vectors to find the solution of system of ordinary differential equations 8

$$\frac{dy_1}{dx_1} = 5y_1 + 2y_2$$

$$\frac{dy_2}{dx_1} = 4y_1 + y_2$$

7 Show that in the interval (0,1) 8

$$\cos\pi x = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{n}{4n^2-1}\sin 2n\pi x$$

8 Find $f(x)$ whose Fourier cosine transform is $\sin\frac{ax}{x}$ 8

9 Find the Z –transform of (i) n^2 (ii) $n\sin n\theta$ 8

10 Using Z-transform solve the difference equation 8

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n$$

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