Reg. No.:

Name :

VIT BHOPAL

MID TERM EXAMINATIONS – April 2024

Programme	:	B.Tech.	Semester	:	Winter 2023-24
Course Title	:	Discrete Mathematics and Graph Theory /	Slot	:	B21+B22+B23
Course Code	:	MAT2002	Max. Marks	:	50
Time	:	1 ½ hours			

Answer all the Questions

Q.No.	Sub. Sec.	Question Description	Marks
1	(a)	Consider the set F of all functions from $\mathbb{Z}$ to $\mathbb{Z}$, i. e., $F = \{f \mid f: \mathbb{Z} \to \mathbb{Z}\}$, where $\mathbb{Z}$ is the set of integers. Which of the relations, given below, on F are equivalence relations? Determine the properties of an equivalence relation that the others lack. (i) $R_1 = \{(f,g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$ (ii) $R_2 = \{(f,g) \mid f(0) = g(1) \text{ and } f(1) = g(0)\}$ (iii) $R_3 = \{(f,g) \mid f(x) - g(x) = 1 \text{ for all } x \in \mathbb{Z}\}$.	6
	(b)	Define two equivalence relations on the set of students in your discrete mathematics class different from the relations discussed in the class. Determine the equivalence classes for each of these equivalence relations.	4
2	(a)	Give a poset that has (i) a maximal element but no minimal element. (ii) neither a maximal nor a minimal element.	4
	(b)	Give an example of an infinite lattice with (i) neither a least nor a greatest element. (ii) a least but not a greatest element. (iii) a greatest but not a least element.	6
3		Without using the truth table, show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.	10
4	(a)	Write the contrapositive, the converse, and the inverse of the statement: "If two graphs G and H are isomorphic, then they have the same order, same size, and the degrees of the vertices of G are the same as the degrees of the vertices of H."	6
	(b)	(i) Using De Morgan's laws, prove the logical equivalence of the statements: "For all real number x, if x is not equal to 0, then $\frac{1}{x}$ is defined." "There exists a real number x such that x is equal to 0 or $\frac{1}{x}$ is not defined."	4
		(ii) Use De Morgan's laws to find the negative of the statement: "She is exquisite and tall and I like her."	
5		Draw all simple non-isomorphic graphs with four vertices.	10

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