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TERM END EXAMINATIONS (TEE) – May 2024

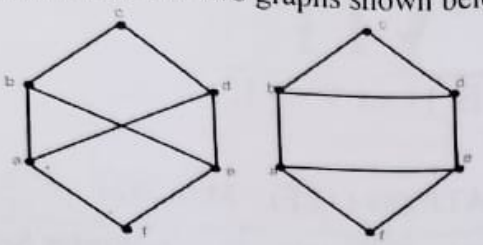
Programme	:	B.Tech.	Semester	:	Winter Semester 2023-2024
Course Title/ Course Code	:	Discrete Mathematics and Graph Theory/MAT2002	Slot	:	D11+D12+D13
Time	:	3 Hrs.	Max. Marks	:	100

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	Let R_5 be the relation on the set Z of integers, defined by $x \equiv y \pmod 5$ which reads as "x is congruent to y modulo 5 "i.e. x-y is divisible by 5. Prove that R_5 is an equivalence relation.	6
	(b)	Convert CNF to DNF $f(x,y,z) = (x' + y + z')(x' + y + z)(x + y' + z)$	6
		OR	
	(c)	Check whether the function $f: R \to R$ defined by $f(x) = 2x+3$, for all x belongs to R is invertible .Where R is a set of Real numbers.	6
	(d)	Draw the Hasse Diagram of Poset $\{S, \subseteq\}$, where S= {a, b, c}. Also find the greatest element, least element, maximal element and minimal element.	6
2	(a)	(i) Show that $\{[(p \vee q) \Rightarrow r] \wedge (\sim p)\} \Rightarrow (q \Rightarrow r)$ is a tautology without using the Truth Table.	6
		(ii) Determine by rules of inference, whether the following argument is valid. "If n is a real number with n>3 then n^2>9. Suppose $n^2 \leq 9. Then\ n \leq 3$."	6
		OR	
	(b)	(i) Obtain Disjunctive Normal Form of the following; $p \Rightarrow ((p \Rightarrow q) \wedge \sim (\sim q \vee \sim p))$	6
		(ii) Express the following statement in symbolic form. Also write the "Converse, Contrapositive and Negation" of the statement. "If 9 is odd then the square of 9 is odd."	6
3	(a)	Prove that the number of vertices of odd degree in a graph G is always even.	6
	(b)	Does 3-regular graph on 14 vertices exist? If yes, draw it. What about on 17 vertices?	6
		OR	

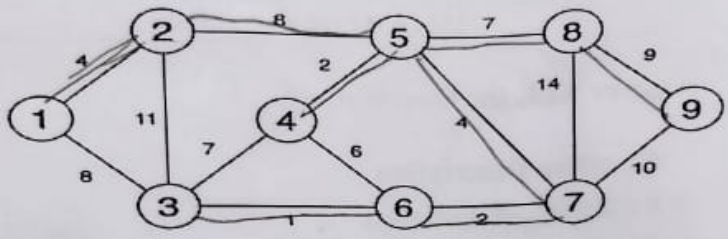
(c) Prove that a simple complete graph with n - vertices has the {n(n-1)/2} edges. 6

(d) Discuss whether the two graphs shown below are isomorphic? 6



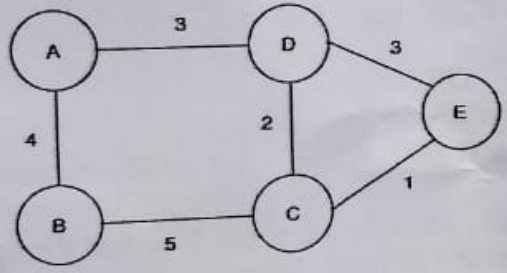
Find the Minimum Spanning Tree by using Kruskal's algorithm of the following graph. 8

(a)



Draw all possible Spanning Tree of the adjacent graph. 4

(b)



4

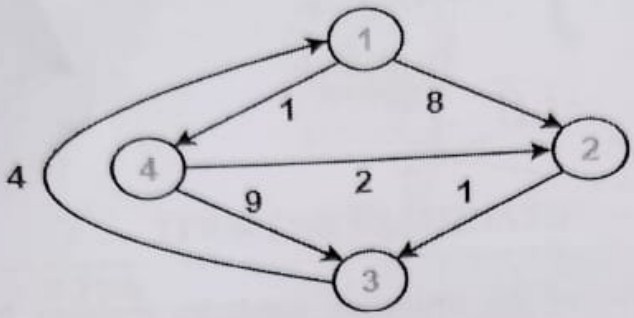
OR

A Graph G has the following adjacency matrix. Check wether it is connected. 12
(Without constucting the graph).

(c)

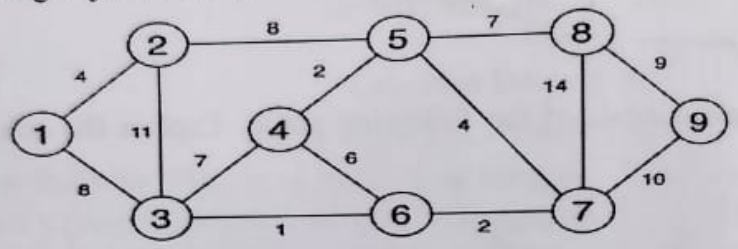
$$X(G)=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

5 (a) Discuss the process of finding the shortest path for each vertex pair of weighted directed graph by using Floyd- Warshall Algorithm. Find the shortest path for each vertex pair of following weighted directed graph by using Floyd-Warshall Algorithm 12

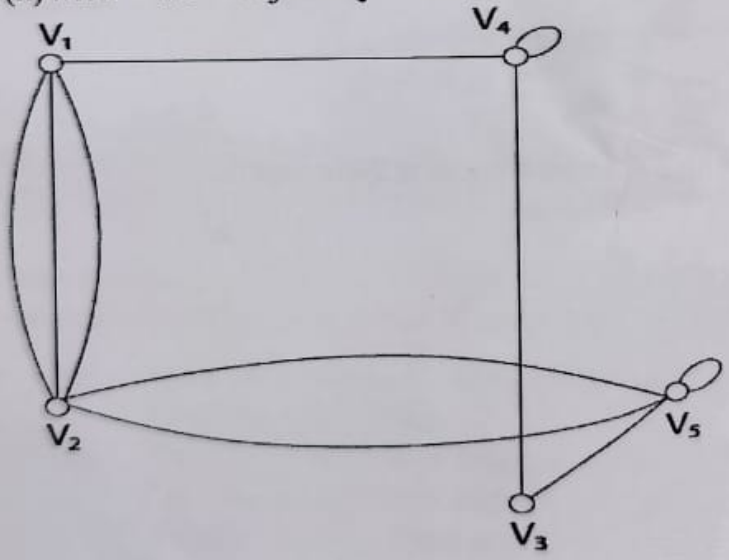


OR

(b) (i)Explain Dijkstra's Algorithm. Find the shortest path from vertex 1 to vertex 9 by using Dijkstra's Algorithm. 8



(ii)Write the adjacency Matrix of the multi graph shown below. 4



PART B – (40 Marks)

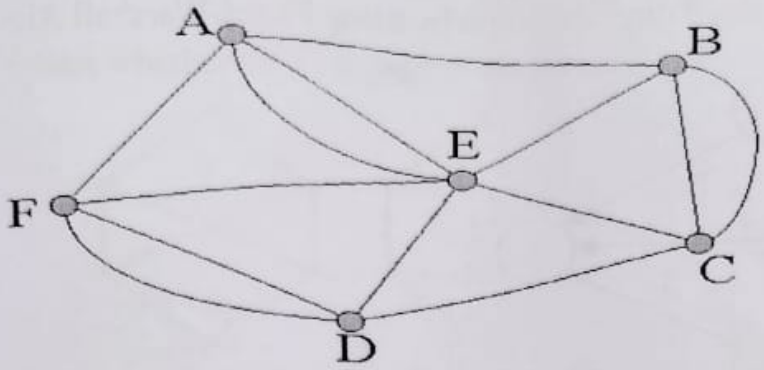
6 Let S= {3,5,9,15,24,45} be the set, then prove that the relation " | " (means divides) defined on S is a Partial order relation. 8
Also find Minimal, Maximal, Least and Greatest elements of the Poset (S, |).

7 Check the following for Tautology, Contradiction or Contingency using the Truth Table. 4+4

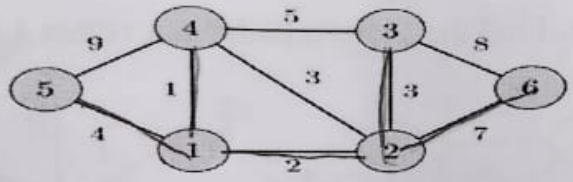
(i)$(p \vee \sim q) \Rightarrow (p \wedge q)$

(ii) $(p \Rightarrow q) \Leftrightarrow (p \wedge \sim q)$

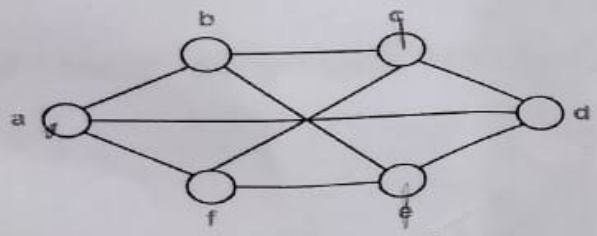
8 Is the graph shown below a Euler Circuit? If yes, draw the Euler Circuit with proper direction. 8



9 Find the Minimal Spanning Tree of the following graph by applying Prim's algorithm. 8



10 (i) What is the chromatic number of the following graph. Explain the colouring procedure. 4+



(ii) Prove that every Tree with 2 or more vertices is 2-chromatic.

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