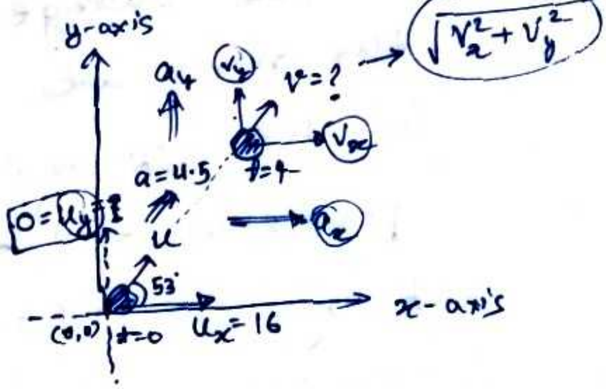
PHY 1001 MID-TERM EXAM - DEC - 2022 (P1)

(1) NLM

A particle moves in x-y plane with a constant acceleration of $4.5\,m/s^2$ in the direction making an angle of 53° with the x-axis. At $t=0$ sec the particle is at the origin and its velocity is 16 m/s along the x-axis. Find the velocity and position of the particle at $t=4$ sec.
(where, $\cos 53^\circ = 3/5$ and $\sin 53^\circ = 4/5$)

$\boxed{\text{10 marks}}$

Sol^n:-



Along x-axis

$a_x = a\cos 53^\circ = 4.5 \times \frac{3}{5}$

$a_x = 2.7\,m/s^2$

$v_x = u_x + a_x t = 16 + 2.7 \times 4 = 26.8\,m/s$

Along y-axis

$a_y = a\sin 53^\circ = 4.5 \times \frac{4}{5} = 3.6\,m/s^2$

$v_y = \cancel{u_y}^{\,0} + a_y t = 0 + 3.6 \times 4 = 14.4\,m/s$

$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{(26.8)^2 + (14.4)^2} = 30.42\,m/s$ Ans

Now for getting the position at $t=4$ sec

$s = ut + \frac{1}{2}at^2$

$\therefore S_x = u_x t + \frac{1}{2}a_x t^2 = 16 \times 4 + \frac{1}{2} \times 2.7 \times (4)^2 = 85.6\,m$

$S_y = u_y t + \frac{1}{2}a_y t^2 = 0 \times 4 + \frac{1}{2} \times 3.6 \times (4)^2 = 28.8\,m$ A

or so, position (85.6, 28.8) A

(2) (a) A thin cylindrical wheel of radius $r = 75$ cm is allowed to spin on a frictionless axle. The wheel, which is initially at rest, has a tangential force applied at right angles to its radius of magnitude 100N. The wheel have a moment of interia which is equal to 3×10^4 gm^2, find —

a) The torque on wheel.

b) The angular acceleration of the wheel.

c) The angular velocity of the wheel at 6 sec.

$\boxed{\text{5 marks}}$

sol^n:- a) $\tau = F \cdot r_\perp = \frac{100 \times 75}{100} = 75$ Nm

b) α = angular acc,

$\tau = I\alpha \quad \therefore \alpha = \frac{\tau}{I} = \frac{75}{3 \times 10^4}$

silly mistake be careful → $\alpha = 25 \times 10^{-4}$

we have to convert into kg → $I = 3 \times 10^4$ g m^2

$= \frac{3 \times 10^4 \text{ kg m}^2}{1000}$

$I = 3 \times 10 = 30$ kg m^2

Now $\alpha = \frac{75}{30}$

$\therefore \alpha = 2.5$ rad/s^2

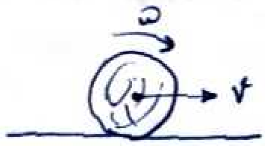
c) $\omega = \cancel{\omega_0}^{0} + \alpha t$

$\omega = 2.5 \times 6 = 15$ rad/s

(2) <b> A hollow sphere of mass M and radius R rolls on a Horizontal surface such that center of sphere moves with speed v. Calculate the kinetic energy and angular momentum of the sphere? (MOI of hollow sphere, $I_c = \frac{2MR^2}{3}$)

$\boxed{5 \text{ marks}}$

Sol^n:-



So. $KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$$\left\{ \begin{array}{l} v = R\omega \\ \therefore \omega = \frac{v}{R} \end{array} \right\}$$

$$KE = \frac{1}{2}Mv^2 + \frac{1}{\cancel{2}} \times \frac{\cancel{2}MR^2}{3} \times \frac{v^2}{R^2}$$

$$KE = \frac{Mv^2}{2} + \frac{Mv^2}{3}$$

$$KE = \frac{5Mv^2}{6}$$

Angular momentum, $L = I\omega$

$$L = \frac{2MR^2}{3} \times \frac{v}{R}$$

$$L = \frac{2MvR}{3}$$

*** (3) A particle of mass 'm' is trapped inside an infinite potential box of length L ($0<x<L$). Find the normalized wave function for third excited state, also find the energy difference between 5^{th} energy state and ground state.

$\boxed{\text{10 marks}}$

Soln:- we know that,
normalized wave fcn for n^{th} excited state is -

$$\psi_n(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi x}{L}\right)$$

$\because$ 3^{rd} excited state that means $n=4$

$$\therefore \psi_n(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{4\pi x}{L}\right)$$ A

Again, we know that,

The Energy of n^{th} energy state is —

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Here, $\hbar$ = reduced planck's const.
m = mass of the particle
L = length of the box
n = quantum number.

$$\therefore E_5 = \frac{5^2\pi^2\hbar^2}{2mL^2}$$

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2}$$

$\therefore$ Energy difference,

$$\Delta E = E_5 - E_1$$

$$\Delta E = \frac{25\pi^2\hbar^2}{2mL^2} - \frac{\pi^2\hbar^2}{2mL^2}$$

$$\Delta E = \frac{\cancel{24}\pi^2\hbar^2}{\cancel{2}mL^2} = 12\,\frac{\pi^2\hbar^2}{mL^2}$$ A

(4) Explain your thoughts on wave particle duality. Briefly discuss the significance of wave function in quantum mechanics with its properties.

$\boxed{\text{10 marks}}$

sol:- Wave particle duality suggest that the particles have ~~charestates~~ characteristics of both (interference, diffraction) wave and (localized energy, quantized properties) particles.

This is is manifestation of complementory or, a concept introduced by Niels Bohr. It implies that different experimental setups reveal different aspects of a particle's behavior.

Here, are some significance of the wave fun -

i) The wave function (ψ) is a mathematical term in quantum mech. which gives the information about probability distribution of a particle's position.

ii) The wave function contain all the information about a quantum system, including its energy, momentum, and angular momentum.

~~iii~~

and. Here are few property ___

i) Normalization - The integral of the square of the wave function over all space must be equal to 1, which ensure that the particle is somewhere in space.

ii) Superposition - quantum particles can exist in multiple states simultaneously due to the principle of superposition,

iii) The wave fun. describes the inherent uncertainty in the position and momentum of a particle, as described by Heisenberg's uncertainty principle.

iv) The wave fun must satisfy specific boundary conditions, especially in the case of particles in confined systems like potential wells.

(5) How does nanomaterial's differ from bulk materials? Explain in terms of surface to volume ratio and briefly explain quantum confinement effect.

10 marks

Soln:- Nanomaterials ~~are~~ differ from bulk materials primarily in their size and the unique properties that arise due to their nanoscale dimensions.

The key difference between nanomaterials and bulk materials can be explained in terms of surface to volume ratio and the quantum confinement effect.

So, Surface to volume ratio :-

Nanomaterials have a significantly higher surface to volume ratio compared to bulk materials.

So, as the size of a material is reduced to the nanoscales then surface ~~to~~ area to volume ratio is increasing.

This high surface to volume ratio in nanomaterials makes their surface properties dominant and plays a crucial role in determining their behavior and reactivity. Surface atoms can interact more readily with their environment.

And Quantum confinement effect is —
a phenomenon that occurs when particles are confined within nanoscale dimensions. It arises due to confinement (fix) of electron and other charge carries within a small space.

Quantum confinement leads to shift in a energy band structure of Nanomaterials. for eg:- in semiconductor, nanoparticles. the energy gap between the valence and conduction bands incr. as the particle size decreases, so that the conductivity of semiconductor decreases.

And causing change in optical and electronic properties. the effect can result in change the colours, and electronic behaviors. etc.